Lectures 10-11: Planetary interiors

o Topics to be covered:

o Heat of formation

o Chemical differentiation

o Natural radioactivity

o Planet cooling

Surface of Venus

Summary of planetary interiors

o   Make-up of planetary interiors is dominated by physics of materials under high temperatures and pressures.

o   Starting with cold, low pressure regions, rocky materials are solids.

o   As one goes deeper into a planet, temperature and pressures rise. Solids become semi-solid, plastic-like materials.

o   With higher temperatures and pressures, semi-solids become liquids. With even higher temperatures and pressures liquid or molten rocky materials undergo a phase change and become solids again.

    o   This is why the very inner cores of the Earth and Venus are solid, surrounded by liquid outer cores.
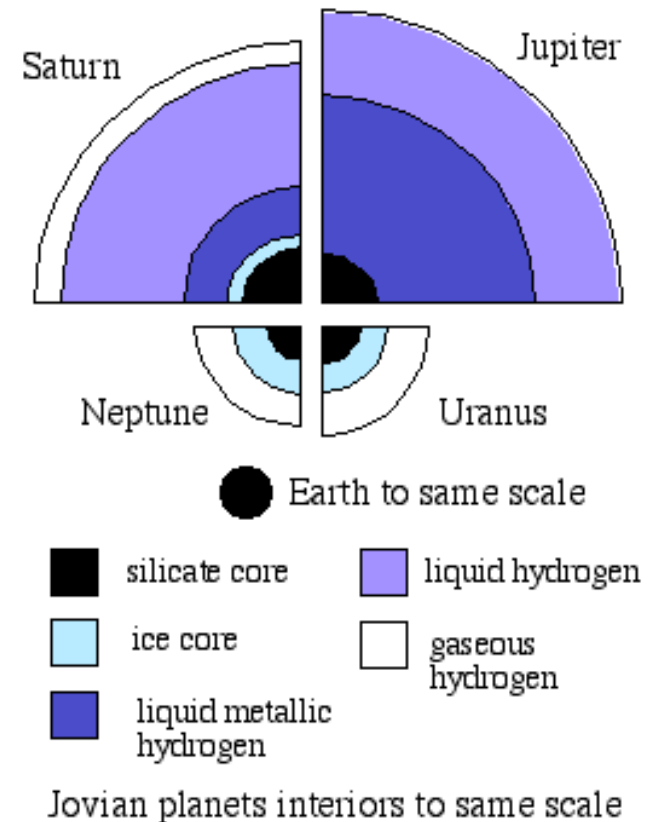
Summary of terrestrial interiors

o *Mercury* has large iron core about 3500 km in diameter which is ~60% of its total mass, surrounded by a silicate layer ~700 km thick. Its core is probably partially molten.

o *Mars* has a solid Fe and/or iron-sulfide core ~2600-4000 km in diameter, surrounded by a silicate mantle and rocky crust that is probably several hundred km thick.

o *Venus'* interior is like the Earth's, except its Fe-Ni core probably makes up a smaller percentage of its interior.

Venus   Earth

Mercury   Mars

solid iron core	silicate mantle
liquid iron core	silicate crust
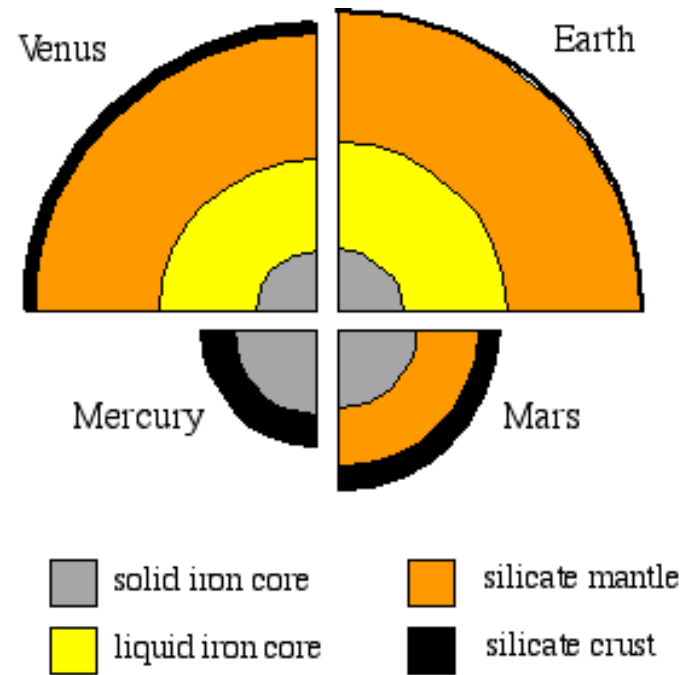
Terrestrial planet interiors to same scale

Summary of jovian interiors

o Jupiter's H/He atmosphere is ~1,000 km thick and merges smoothly with the layer of liquid molecular H, which is ~20,000-21,000 km thick. Pressure near center is sufficient to create a liquid metallic H layer ~37,000-38,000 km thick. Probably has silicate/ice core twice diameter of Earth with ~14 times Earth's mass.

o Saturn is smaller version of Jupiter: silicate core ~26000 km in diameter, ice layer about 3500 km thick, beneath a ~12,000 km thick layer of liquid metallic H. Then liquid molecular H layer around 28,000 kilometers thick, and atmosphere about 2000 km thick.

o Compression on Uranus/Neptune probably not enough to liquefy H. Uranus/Neptune have silicate cores ~8000-8500 km in diameter surrounded by a slushy mantle of water mixed with ammonia and methane ~7000-8000 kilometers thick. At top is a 9000 -10000 km thick atmosphere of H and He.

Saturn   Jupiter

Neptune   Uranus

● Earth to same scale

■ silicate core   ■ liquid hydrogen

□ ice core   □ gaseous hydrogen

■ liquid metallic hydrogen

Jovian planets interiors to same scale

Heat of formation

o   Initial planet internal temperature is due to accretion.

o   Conservation of energy tells us:

    o   Potential energy of material is converted into kinetic energy of motion.

    o   Upon hitting the planet, the kinetic energy of motion is converted into internal heat energy of the planet - hence initial hot phase expected.
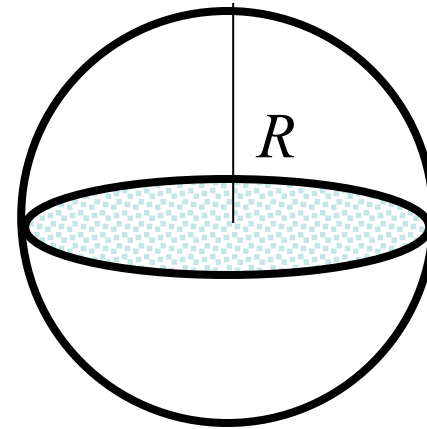
R
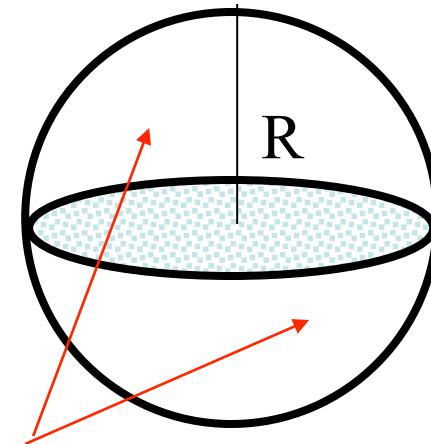
Two hemispheres with centers separated by distance R

Heat of formation

o Consider planet made of two hemispheres of radius R and mass $M/2$.

o The potential energy released, ΔU, by expanding these two halves to infinity will be

$$\Delta U \approx \frac{G(M/2)^2}{R}$$

o Conservation of energy says that the same amount of energy will be liberated in the reverse process =>

$$E_{heat} = \Delta U$$

Heat of formation

o Amount of energy (dU) gained by addition of dm to body of mass M:

$$dU = \frac{GM}{r}dm$$

o If body is spherical, $M = 4/3 \ \pi\rho r^3$

o Addition of mass $dm => dm = 4 \ \pi\rho \ r^2 \ dr$

o Substituting for M and dm into equation for dU and integrating gives:

$$U = \int_0^R 3G\left(\frac{4}{3}\pi\rho\right)^2 r^4 dr$$

$$= \frac{3}{5}G\left(\frac{4}{3}\pi\rho\right)^2 R^5$$

$$= \frac{3}{5}\frac{GM^2}{R}$$

o This is total energy going into body from accretion.

Heat of formation

o Must consider specific heat capacity C_P of a planet. Defined as amount of heat energy required to raise unit mass of material by 1K.

    o Rock: $C_P = 1200$ J / kg / K

    o Nickel: $C_P = 460$ J / kg / K

    o Ice: $C_P = 2100$ J / kg / K

o So, change ΔT in temperature of a mass M due to E_{heat} is: $E_{heat} = C_P M \Delta T$

o Equating this to energy due to accretion: $C_P M \Delta T = \dfrac{3}{5} \dfrac{GM^2}{R}$

o Rearranging gives

$$\boxed{\Delta T = \frac{3}{5} \frac{GM}{C_p R}}$$
         Eqn (*)

o This is the maximum temperature of a planetary interior that results from accretion.

Consequences

o Previous can be written

$$\Delta T = \frac{4G\pi\rho}{5C_P} R_p^2 = 6.28 \times 10^{-10} R_p^2$$

o ΔT for the Earth is large (40,000 K) and much greater than the melting temperature of rock (~1200 K).

o Setting $\Delta T \sim 1200$ K (melting temperature of rock) => R~1000 km

=> Objects bigger than large asteroids melt during accretion

=> Earth was a molten blob of rock

Heat of formation

o   Maximum temperatures attainable by accretion from infinity:

Object	Radius (km)	Mass (kg)	ΔT (K)
Earth	6,378	6×10^{24}	30,000
Venus	6,062	5×10^{24}	?
Mars	3,397	6×10^{23}	6,000

o   Did this energy dissipate at about the same rate as it was generated or did it build up quicker than it could be dissipated?

o   For Earth think still some remnant heat of formation. For smaller planets, heat of formation may have been dissipated as quickly as planet formed.
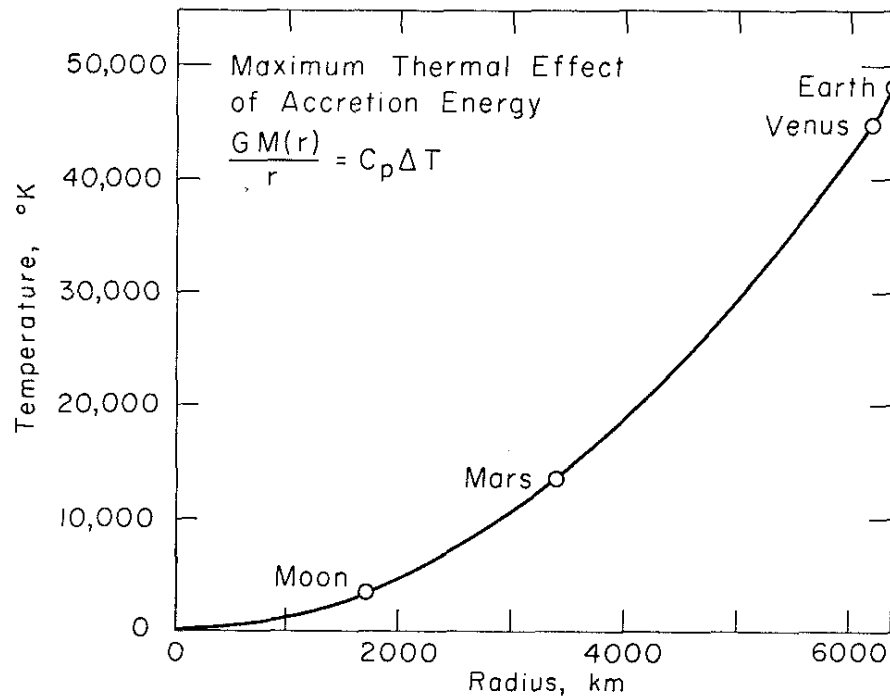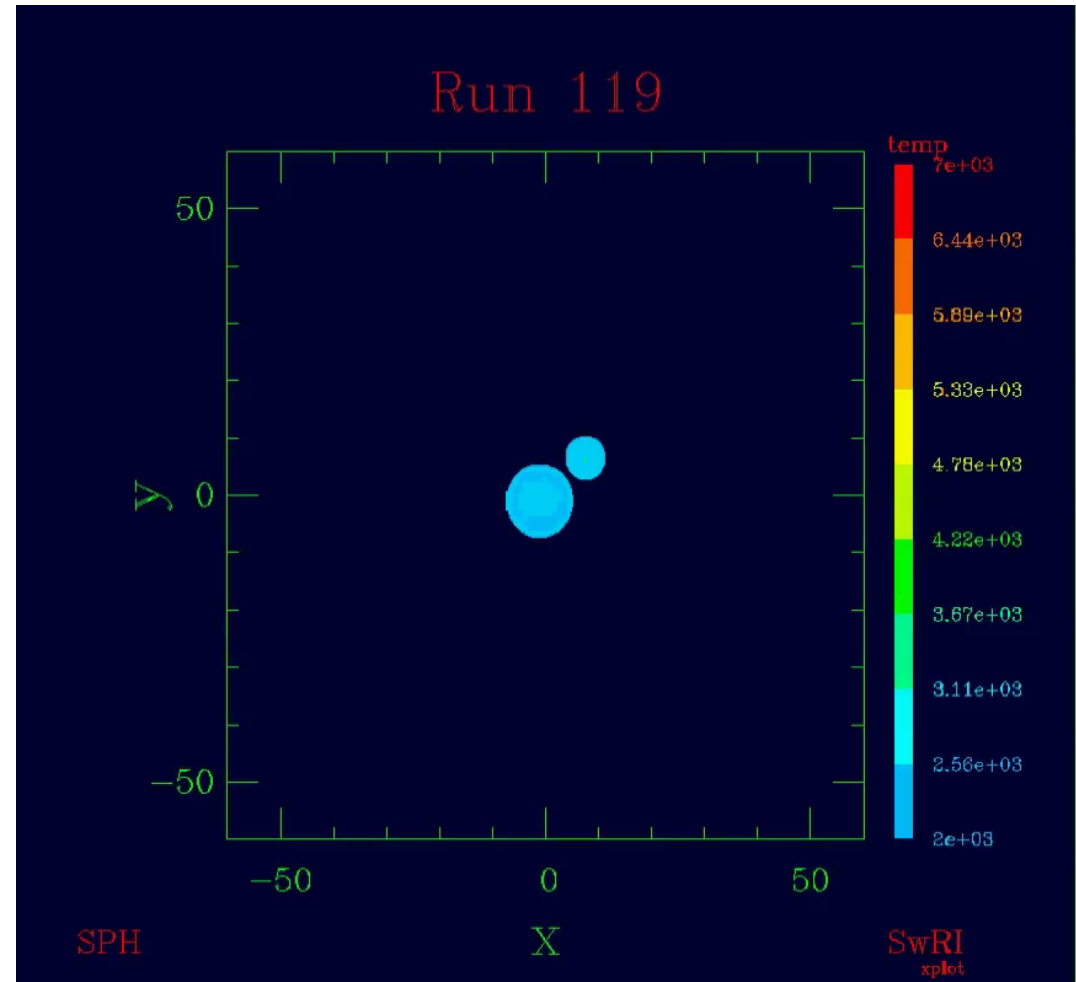
Heat of formation

Fig. 1.   Maximum surface temperatures from gravitational accretion.

o   From Hanks & Anderson (1969).

o   http://www.gps.caltech.edu/uploads/File/People/dla/DLApepi69.pdf

Temperature of formation

o Simulation of impact between 80% M_E proto-Earth and 10% M_E object (Canup, R., Icarus, 2004)

o www.boulder.swri.edu/~robin/

Core pressures

o The central pressure is of order the gravitational force between the two hemispheres divided by their area of interaction:

$$P_C = F_{grav} / \pi R^2$$

o Therefore, $P_C = (G/\pi) M^2 / R^4$

o This can be written in terms of the bulk density as

$$P_C = 1.4 \times 10^{-10} \rho^2 R^2 \qquad \text{(Pascals)}$$

o For Jupiter, $<\rho> = 1,300$ kg/m^3 and $R_P = 7 \times 10^7$ m

$$\Rightarrow P_C(\text{Jupiter}) = 1.2 \times 10^{12} \text{ Pa} = 12 \text{ Mega atmospheres}$$

Core pressures

o   Jupiter and Saturn are principally composed of hydrogen.

o   At low temperatures and pressure, H is an insulator in the form of the strongly bound diatomic molecule H_2.

o   At depths of a few thousand kilometers below the upper cloud deck pressure becomes so high that the H_2 becomes dissociated and undergoes a phase transition from the gaseous to a liquid state.

o   For $P > 3$ million atmospheres, atoms are ionised into freely moving protons and electrons. Phase is known as liquid metallic hydrogen (LMH).

o   LMH is highly conducting (the electrons are highly mobile) and this results in the generation of a strong magnetic field.

Chemical differentiation/fractionation

o   After planet formation, planets were homogeneous.

Undifferentiated

o   For bodies with diameters >few km, internal temperatures were large enough to cause partial or total melting of the interiors => allowed materials to separate according to density.

Denser

Lighter

o   "Heavy" materials are those that bond to iron, forming iron-bearing minerals (*siderophiles*, i.e., "iron-lovers"). Siderophiles sink to center of planet, forming core.

Differentiated

o   "Light" materials are those the bond to silicates; called the *lithophiles*. Rise to upper layers of planet.

o   Separation of materials according to density allows a complicated layered structure to form inside a planet.
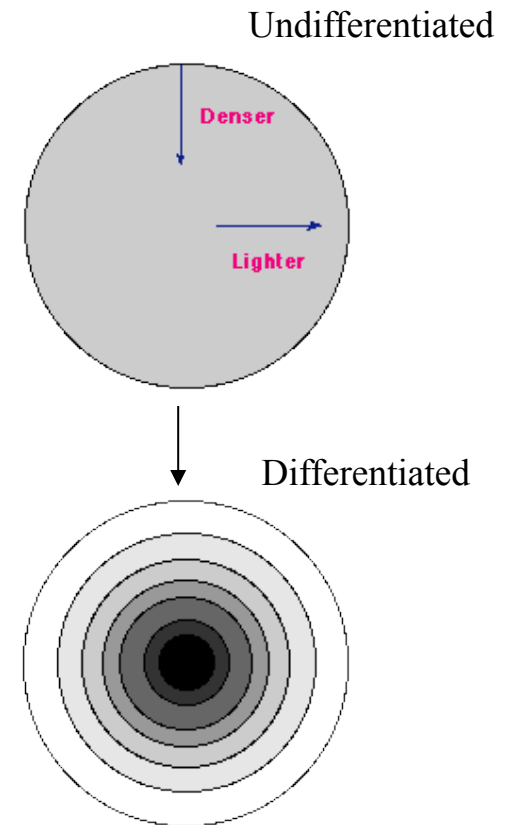
Internal Heating from Core Separation

o Differentiation can be large heat source soon after formation. Will occur when temperature is large enough for metallic iron can sink to core.

o What is gravitational energy released by this process?

o Consider two-components, core ($r < r_c$) and mantle ($r > r_c$), where r_c is core radius.

o For $r < r_c =>$ $M_{r,i} = 4/3\pi r^3 \rho_c$

o For outer regions, $r > r_c =>$

$$M_{r,o} = 4/3\pi r_c^3 \rho_c + 4/3\pi(r^3 - r_c^3)\rho_m$$

$$= \frac{4\pi}{3}\left[(\rho_c - \rho_m)r_c^3 + \rho_m r^3\right]$$

Internal Heating from Core Separation

o Energy released is difference between gravitational energy of homogenous body and body after differentiation:

$$E_g = -G \int_0^{r_c} \frac{M_{r,i} 4\pi r^2 \rho_c}{r} dr + G \int_{r_c}^{R} \frac{M_{r,o} 4\pi r^2 \rho_m}{r} dr$$

$$= -\frac{16\pi^2 G R^5}{15} \left[\rho_c^2 a^5 + \rho_m^2 (1-a^5) + (5/2)\rho_m (\rho_c - \rho_m) a^3 (1-a^2) \right]$$

where $a = r_c / R$.

o By comparison with $E_g = 3GM^2 / 5R$, differentiation releases about one-tenth as much energy as was originally released from accretion.

o See Landstreet "Physical Processes in the Solar System: An introduction to the physics of asteroids, comets, moons and planets"
http://www.astro.uwo.ca/~jlandstr/planets/text/book_asteroids.ps

Chemical differentiation/fractionation

o Large planets (Earth/Venus) are molten long enough for a Fe and Ni core to form.

o Smaller planets (Mars) cool faster and solidify before heavier elements sink to the core => elements like Fe are over abundant in the soil, giving Mars its red color.

o Large planets cool slower, and have thinner crusts. High cooling rates also determine the interior structure.

o Slow cooling rates imply some planets still have warm interiors => more diversified structure (inner core, outer core, semi-solid mantle, etc.)

o Earth is differentiated or layered; highest density in center, lower densities progressively outward
  o Crust - rocky outer layer, brittle (5-40 km)
  o Mantle - solid rocky layer, dense, high pressure, flow
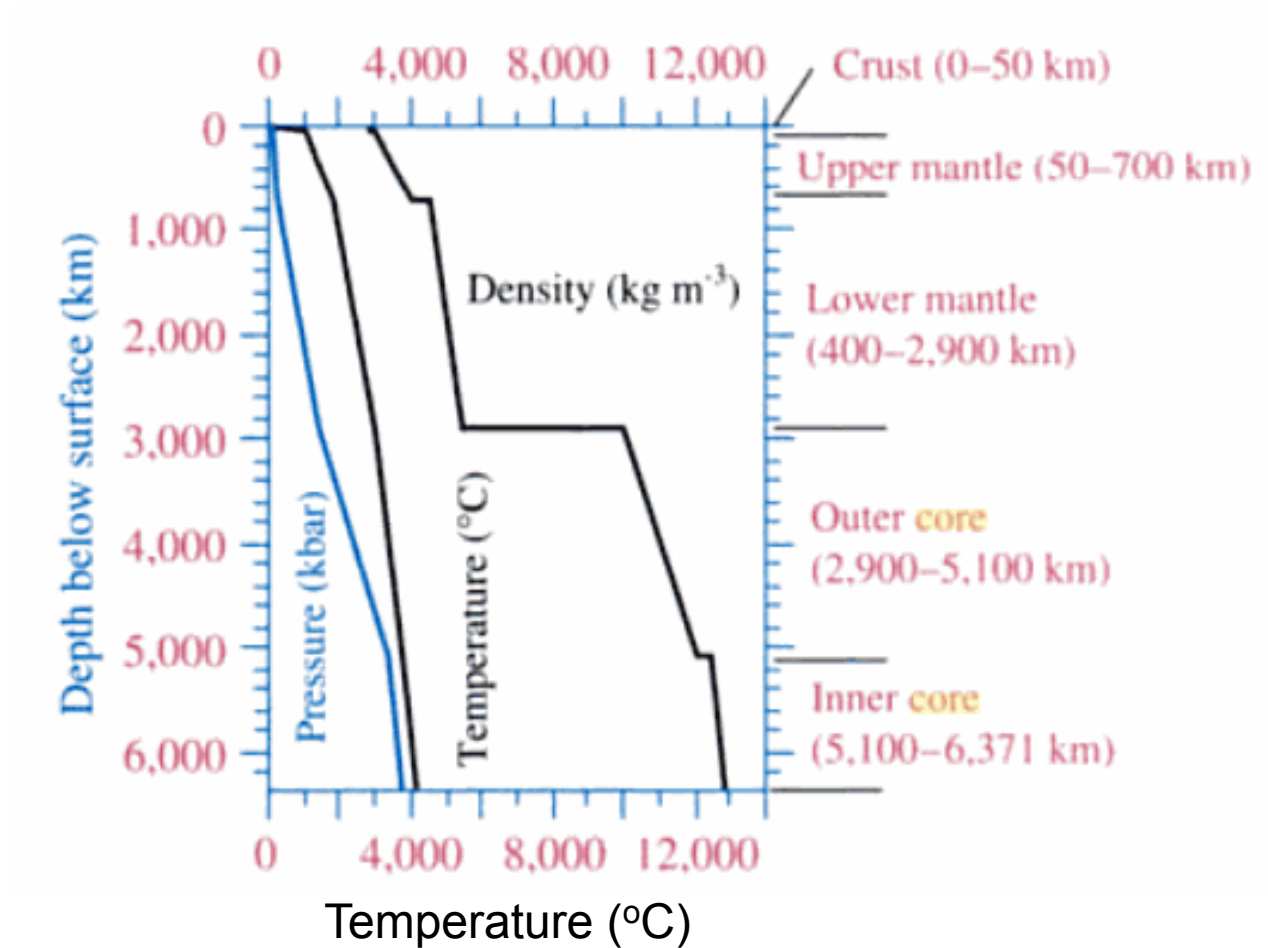 o Outer Core - molten Fe-rich
  o Inner Core - solid Fe, Ni

Differentiation within Earth

o Composed of layers:

  o ***Core:*** Earth central portion. Thickness ~3,470 km and temperature ~6,000K. Pressure so high core is solid.

  o ***Pasty Magna:*** Portion below crust. Divided into mantle (thickness of 1200km) and intermediary layer (called external nucleus, thickness of 1700 km).

  o ***Crust or Lithosphere:*** Earth's external layer. Thickness of 60 km in the mountainous areas and 5 to 10 km in the oceanic basins.

Crust

Lower mantle

D'' layer

Transition region (basaltic magmas source)

Outer core (molten)

Inner core (solid)
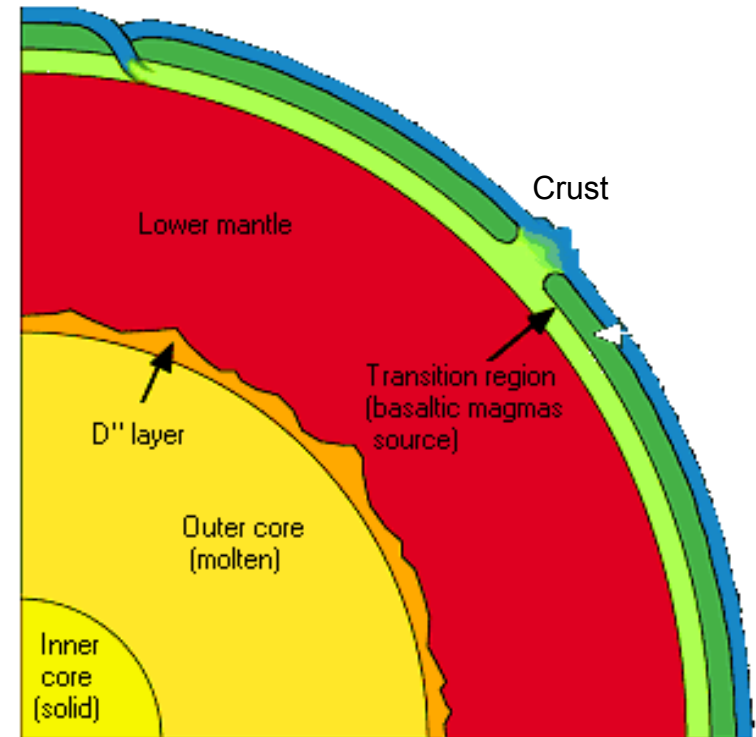
Conditions within Earth

Density (kg/cm^{-3})

# Heating the planets

o Three main sources of heat:

1. Heat of accretion: Generated when planets accreted from planetesimals. Colliding planetesimals convert gravitational potential energy to kinetic energy and then thermal energy.

Gravitational potential energy is converted into kinetic energy.

Kinetic energy is converted into thermal energy.

accretion
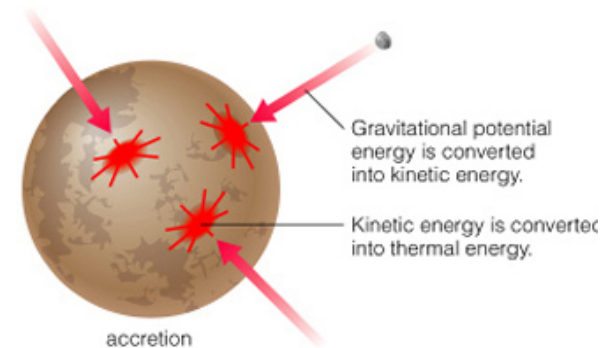
2. Heat of differentiation: Generated at time planets separated into core-mantle-crust. As more dense material sinks, potential energy converted to kinetic and then thermal energy.

Dense materials fall to the core, converting gravitational potential energy into thermal energy

Light materials rise to the surface.
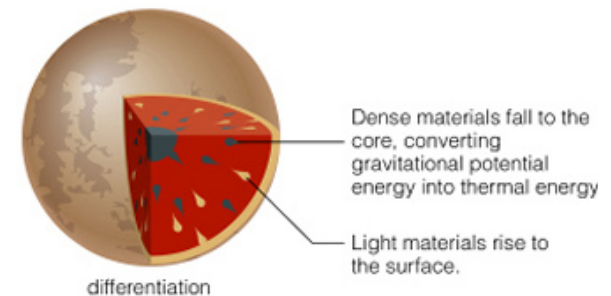
differentiation

3. Heat from radioactive decay: Radioactive nuclei undergo natural decay. Resultant particles collide with neighbouring atoms.

Nuclear energy is converted into thermal energy.
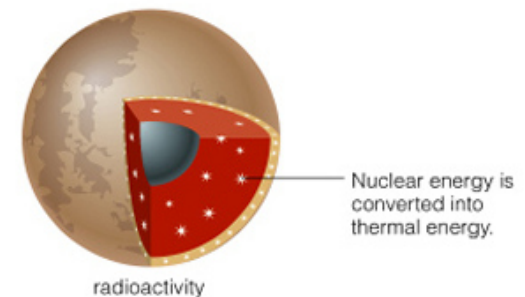
radioactivity

o Accretion and differentiation deposited heat billions of years ago. Radioactive decay is still a source of heat, but was stronger in the past.

Heating by radioactive decay

o   A number of radioactive isotopes occur naturally in rock. As they decay they produce a heating effect.

> o   decay by ejecting e^-, e^+ and α-particles.

o   Some examples:

> o $^{40}K \to {}^{40}Ca + e^- + n$
>
> o   Decay constant: $\lambda(^{40}K) = 5.54 \times 10^{-10}$ yr^{-1} => $t_{1/2} = 0.693 / \lambda = 1.25$ Gyr
>
> o $^{235}U \to {}^{207}Pb + 7\alpha + 4e^-$
>
> o   Decay constant: 9.85×10^{-10} yr^{-1} => $t_{1/2} = 0.6$ Gyr

o   If radioactive decay is a major contributor to heating of planets, elements must have half-lives of ~Gyr.

Heating by radioactive decay

o From Landstreet (page 167)

Table 6.5: Heat released by radioactive decay of elements important in heating planets.

Isotope	Half-life $t_{1/2}$ $(10^9$ yr$)$	Isotope fraction x	Element abundance C	Heating rate (isotope) $(\mathrm{J\,yr^{-1}\,kg^{-1}})$	Heating rate (chondrite) $(\mathrm{J\,yr^{-1}\,kg^{-1}})$
^{40}K	1.25	0.00011	5.60×10^{-4}	9.20×10^2	5.7×10^{-5}
^{87}Rb	50.0	0.293	2.20×10^{-6}	5.44×10^{-1}	3.51×10^{-7}
^{232}Th	13.9	1.00	2.9×10^{-8}	8.37×10^2	2.4×10^{-5}
^{235}U	0.71	0.0072	8.2×10^{-9}	1.80×10^4	1.1×10^{-6}
^{238}U	4.50	0.993	8.2×10^{-9}	2.97×10^3	2.4×10^{-5}

Sources: G.H.A. Cole 1984, *Physics of Planetary Interiors* (Bristol: Adam Hilger Ltd.), Table 5.1. *CRC Handbook of Chemistry and Physics, 1982–83 Ed.* (Boca Raton, Fla: CRC Press, Inc.), B-255. J.T. Wasson 1985, *Meteorites* (New York: W.H. Freeman and Co.), Appendix D.

o See article on Earth's heat budget and geoneutrinos:
http://www.nature.com/ngeo/journal/v4/n9/pdf/ngeo1240.pdf

Heating by radioactive decay

o   If radioactive decay generates heat at a rate Q (J/kg/s), then the total amount of energy generated per second in a uniform spherical planet is:

$$E_{RD} = Q\, M_{planet} = Q\, (4/3\, \pi\, R^3)\, \rho$$

o   Typical values for Q are: $Q\,(^{40}K) = 3.7 \times 10^{-11}$ J/kg/s
$Q\,(^{235}U) = 4.6 \times 10^{-12}$
$Q\,(^{26}Al) = 2.5 \times 10^{-8}$

o   Most energetic radioactive decay process is due to decay of ^{26}Al

$$Q\,(^{26}Al) = 676\, Q\,(^{40}K) = 5435\, Q\,(^{235}U)$$

o   But, $t_{1/2}(^{26}Al) \sim 7 \times 10^5$ yr $=>$ only important soon after planet formed.

Heating by radioactive decay

o   Assume that ^{26}Al can supply energy for t_{Al} (yr).

o   Temperature rise ΔT associated with release of radioactive heat will be

$$E_{RD} = Q\,M\,t_{Al} = M\,C_P\,\Delta T$$
$$\Rightarrow \Delta T = (Q\,/\,C_P)\,t_{Al}$$

o   Typically $t_{Al} \sim 10^6$ years, and $C_P = 2{,}000$ J/kg/s $\Rightarrow \Delta T \sim 400$ K.

o   Total heat liberated in first 1-2 Gyr would have been enough to melt interiors of Earth and Venus.

o   At present, ~50% of Earth's heat output is thought to be due to radioactive heat production. Could have been an order of magnitude larger at time of planet formation.

Heating by radioactive decay

TABLE 5.3

Radioactive Nuclide Heat Production

Nuclide	Abundance (ppm)	Isotope fraction	Decay Rate $\times 10^{-10}/yr$	Energy/Decay $\times 10^{-13} J$	Heat Out $\times 10^{-12} W/kg$ Today	Heat Out $\times 10^{-12} W/kg$ 4.6 $\times 10^9$ yr ago
^{40}K	815.	0.0001167	5.543	1.14	2.9	36.7
^{235}U	0.012	0.0072	9.8485	72.4	0.05	4.6
^{238}U	0.012	0.9928	1.5513	75.9	1.12	2.3
^{232}Th	0.04	1.	0.4948	63.8	1.04	1.3
^{26}Al	10,000.	0.00001	9,500.	3.46	0.	25,000.
^{60}Fe	200,000.	0.014	4,600.	0.2	0.	8,000,000.
^{36}Cl	1,000.	0.015	23,000.	0.34	0.	600,000.

Notes: Abundance columns are elemental and isotope fraction abundances for chondrites; final columns are heat production per kilogram of chondrite. Initial ^{26}Al abundance assumed to be 10^{-5} as inferred from inclusions in the Allende meteorite. Initial abundance of ^{60}Fe assumed to equal present day ^{60}Ni abundance; the ^{36}Cl abundance is 50 ppm, relative to Si.

Radioactive heating of Earth since formation
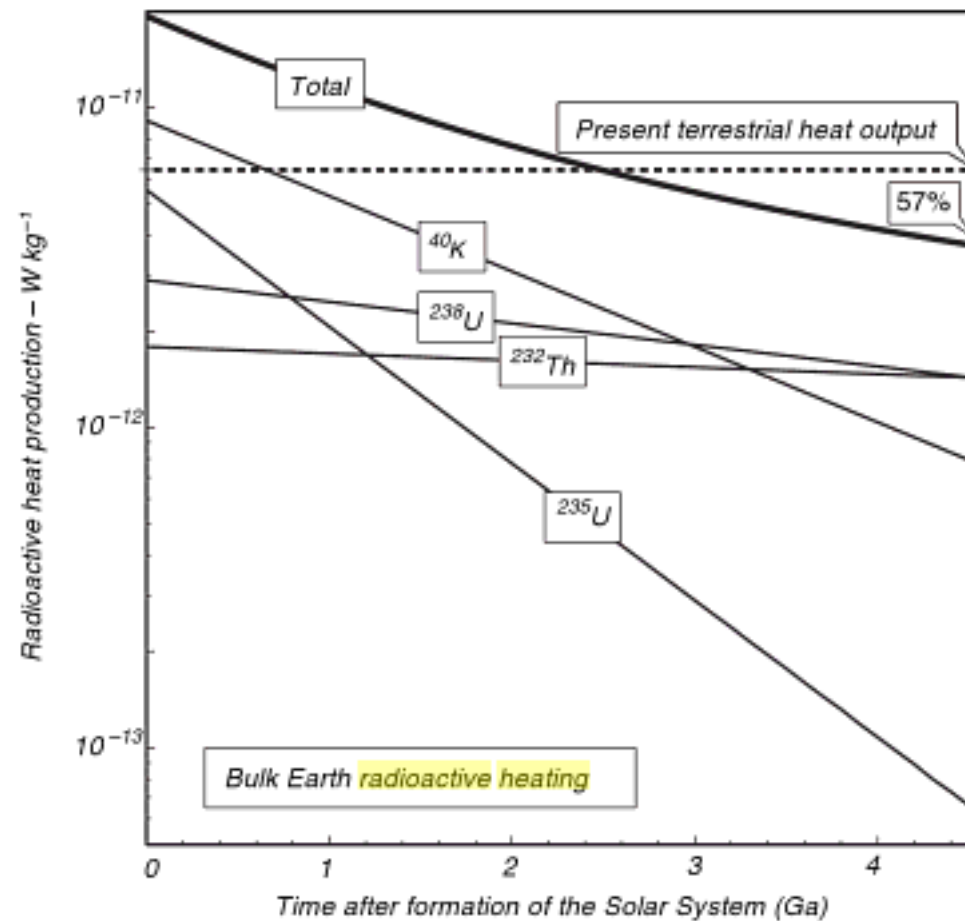
o   From "Thermodynamics of the Earth and Planets" by Alberto Patino Douce

Cooling of the planets

o   Three main cooling mechanisms:

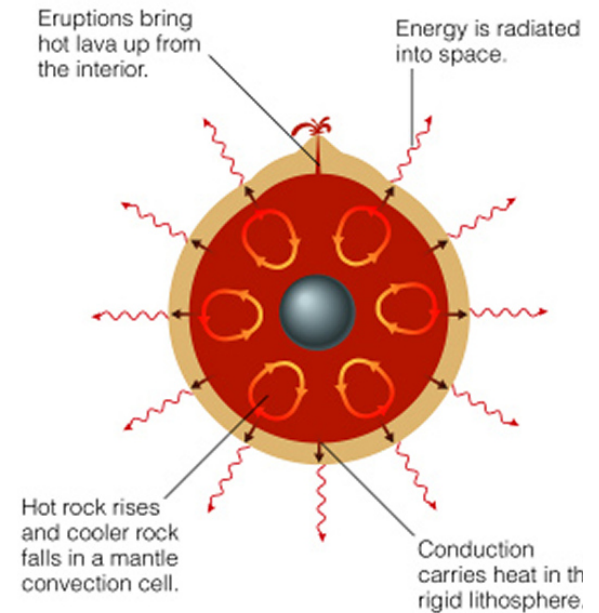1. *Convection* ($Q \sim dT/dr$) in *mantle* carries hot rock to lithosphere where it solidifies.

2. *Conduction* transfers heat from the based of the *lithosphere* to the surface:

$$Q = -k_T \Delta T \quad \text{erg cm}^{-2} \text{ s}^{-1}$$

   wherer k_T is the thermal conductivity.

3. *Radiation* from *surface* transfers heat to space:

$$Q = Area \times \sigma T^4 \quad ergs \ cm^{-2}s^{-1}$$

Cooling timescales

o   Heat content of planet: $\qquad E \sim M\,T \sim 4/3\pi\,R^3\,T$

o   Assume planet cools by radiation: $\quad dE/dt = 4\pi R^2\,\sigma T^4$

o   Therefore, cooling time is: $\quad \tau_{cooling} = \dfrac{E}{dE/dt} \sim \dfrac{R}{T^3}$

o   Implies: 1) Hotter bodies cool faster that cooler bodies and 2) Large bodies take longer to cool.

o   Explains why small bodies cooled early, have large mantles, no magnetic fields, and crater-rich surfaces.

o   *Eg, $R_{Mars}/R_{Earth} \sim 0.5 \Rightarrow$ Mars cooled in half time of Earth. Does this explain Mars' thick crust and lack of plate tectonics, volcanic activity, magnetic field and thin secondary atmosphere?!*

Cooling timescales

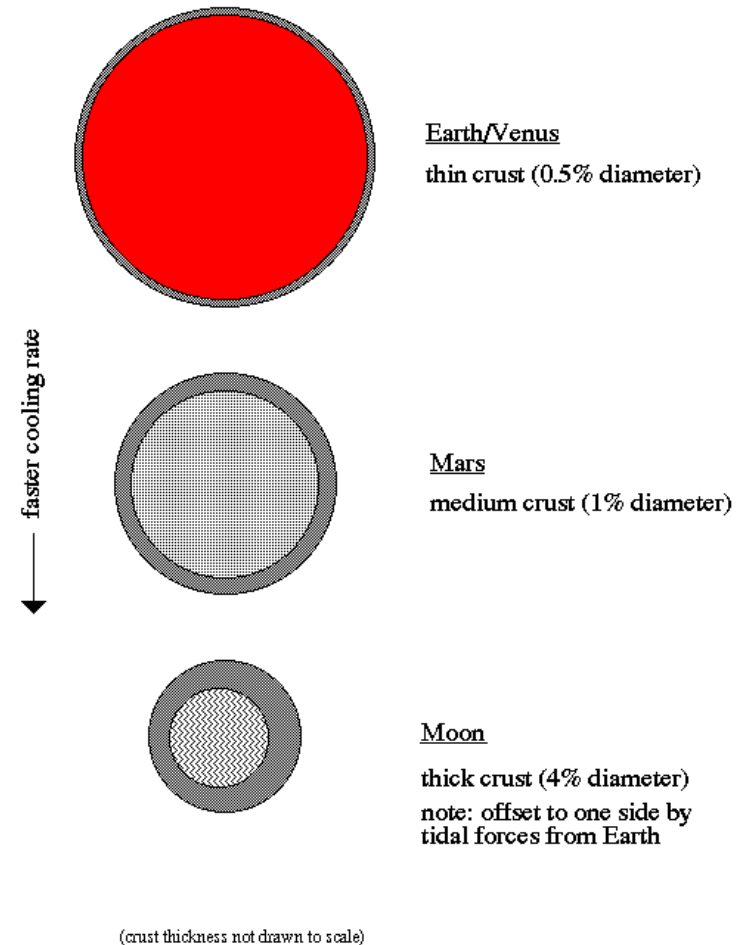o Cooling rate sometimes given in terms of the surface-to-volume ratio:

$$S/V \sim 1/\tau_{cooling}$$

o High S/V for Mercury => high cooling rate => iron core solidified and lowered internal temperature sufficiently to prevent volcanic activity.

o Geological activity is directly related to interior temperature.

o See *Introduction to Planetary Science* by G. Faure and T. M. Mensing.


Surface - to - Volume Ratios of the Terrestrial Planets
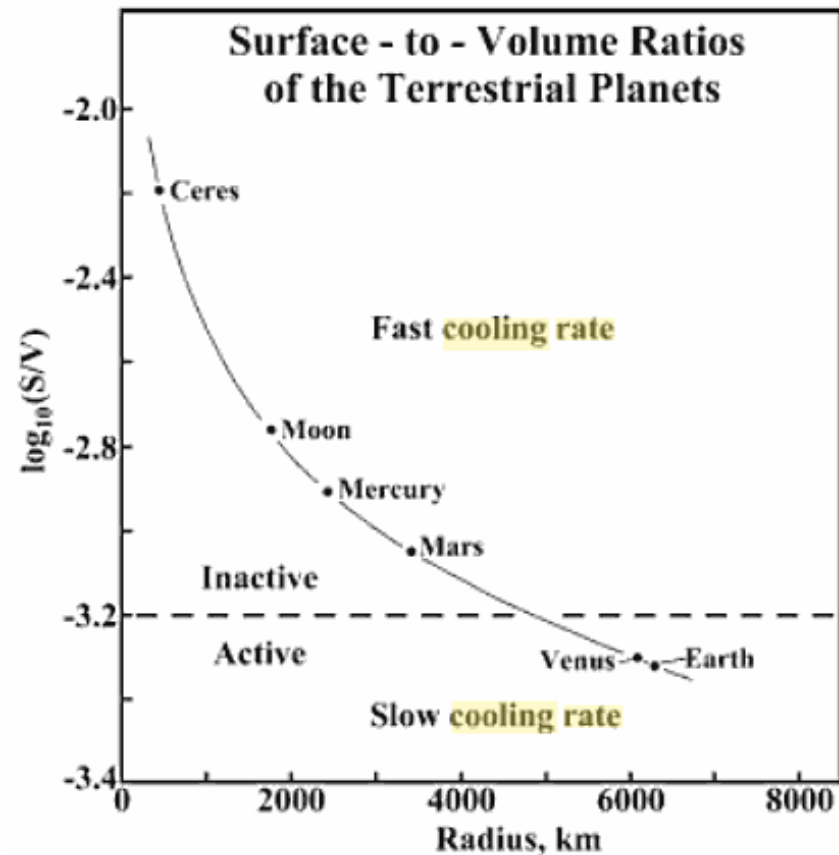
Crust thickness

o Thickness of a planet's crust is determined by rate at which the planet cooled.

o Fast cooling rate (i.e. small planet) results thick crust.

o For major terrestrial planets, crust thickness is proportion to diameter.

o Cooling rate is proportional to total mass of planet. Large planets cool slower, have thinner crusts. High cooling rates also determine the interior structure. Slow cooling rates imply planets that still have warm interiors now.

o A thicker crust also means less tectonic activity.

faster cooling rate

Earth/Venus
thin crust (0.5% diameter)

Mars
medium crust (1% diameter)

Moon
thick crust (4% diameter)
note: offset to one side by tidal forces from Earth

(crust thickness not drawn to scale)

Terrestrial planet structure

o   Small terrestrial planets: Interiors cooled fast.
   o   Heavy cratering.
   o   No volcanoes.
   o   Mercury: uniform cratering.
   o   Moon: highland heavily cratered, lava flows in maria.

o   Medium terrestrial planets: Interior cooled off recently.
   o   Moderate cratering.
   o   Few extinct volcanoes.
   o   E.g., Mars.

o   Large terrestrial planets: Interiors still cooling today.
   o   Light cratering.
   o   Many active volcanoes.
   o   Venus: volcanoes uniformly spread.
   o   Earth: volcanoes at plate boundaries.
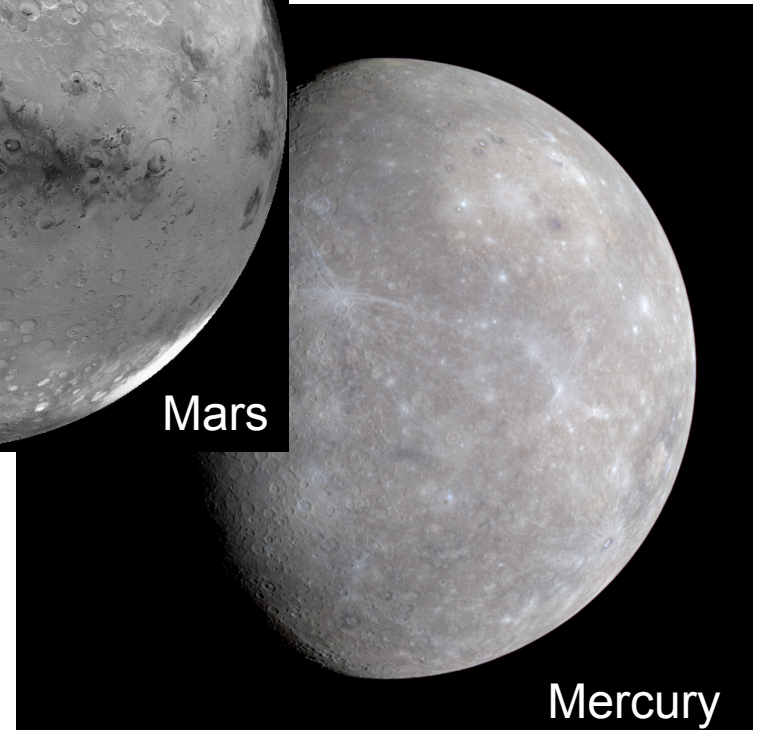

Venus

Earth
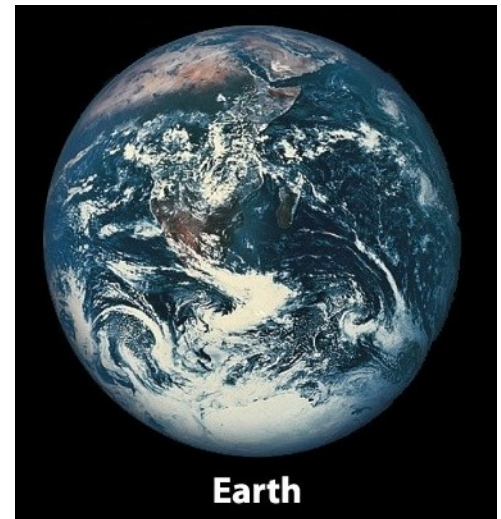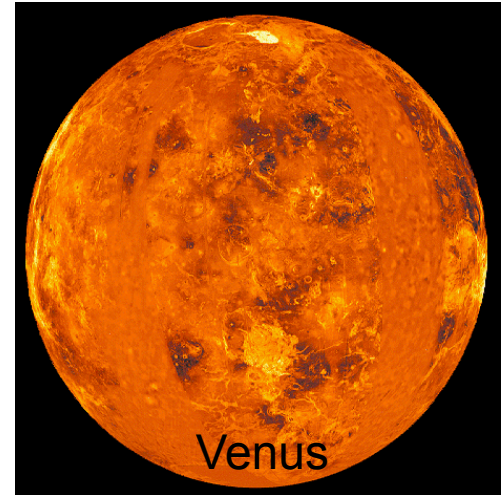
Smaller "terrestrial" objects

The Moon
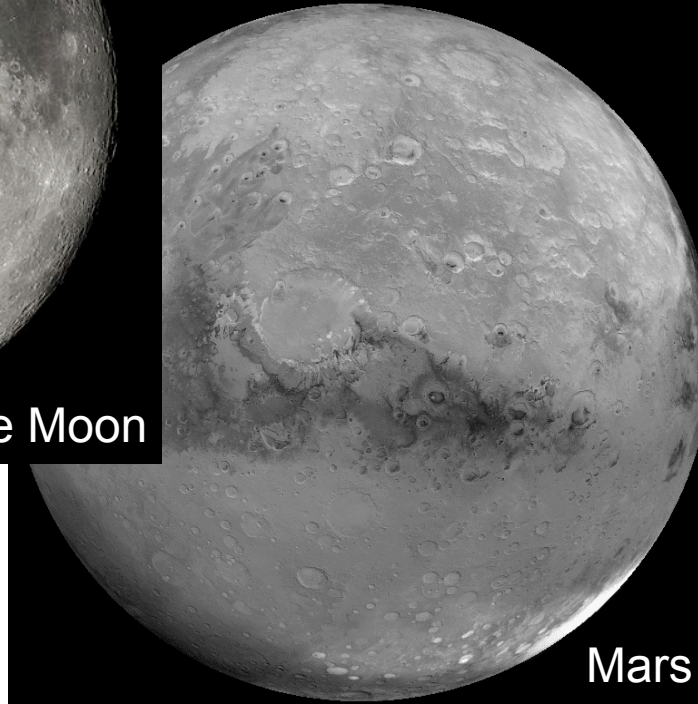
Mars

Mercury

o   Radii < 4,000 km. No tectonic activity, heavily cratered.
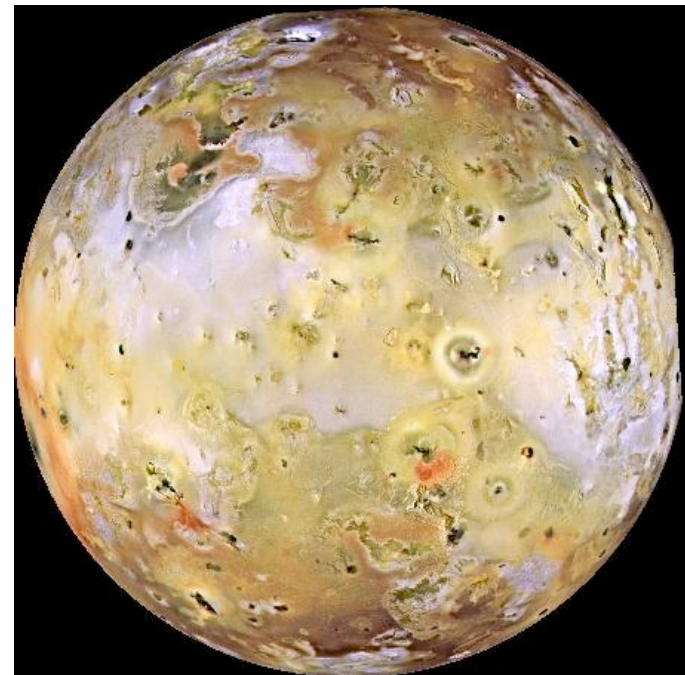
An exception to the rule?

o   Jupiter's moon Io - evidence for lava flows and volcanic activity.

o   Radius is 1,821 km => should therefore have cooled off early and solidified.

o   Interior may be heated by *tidal heating*.

Tidal heating of Io

o   Io is in synchronous orbit around Jupiter it keeps the same face toward Jupiter at all times (just like the Earth's Moon).

o   Differential tidal force of gravity stretches axis of planet along planet-moon line.

o   2:1 resonance orbit with Europa => Io make two orbits for every one orbit that Europa

o   Makes causes constant distortion on Io's shape over its 1.8 days orbit of Jupiter.

2:1
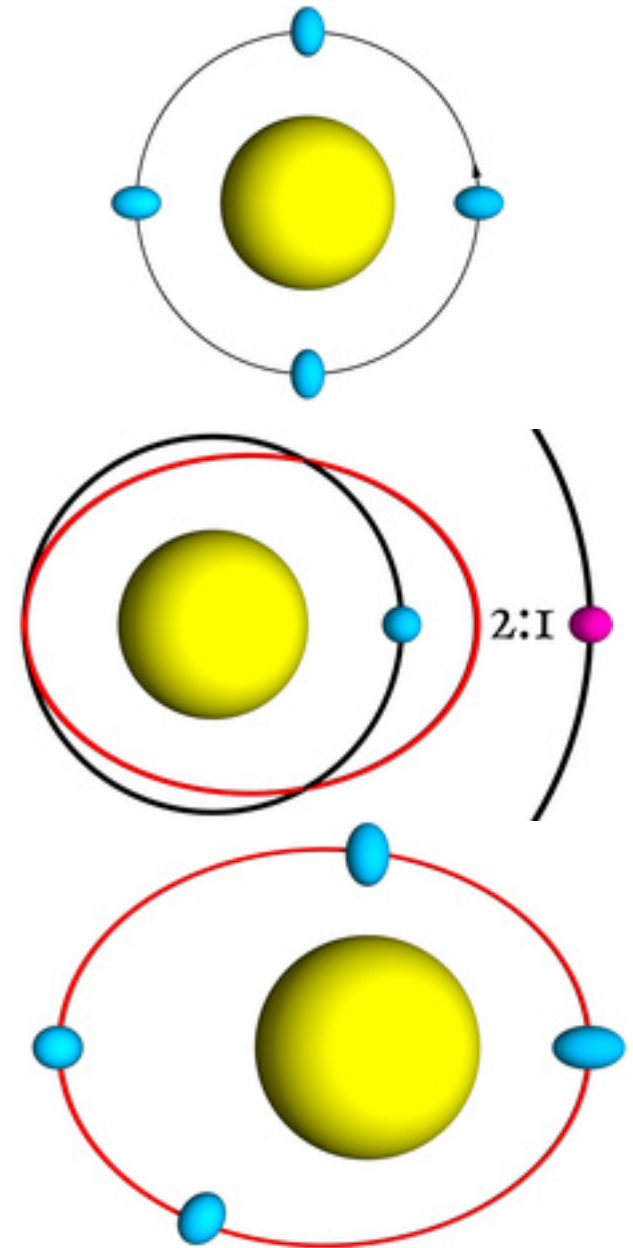
Tidal heating of Io

o   Constant change in shape of Io causes a large amount of friction in interior. This friction generates a great deal of internal heat.
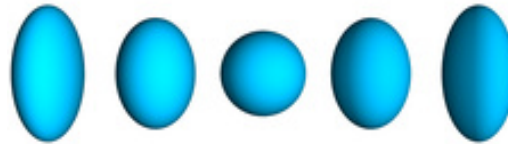
o   This internal heat source drives volcanic activity on the surface of Io. This heating is called **Tidal Heating.**

o   To have Tidal Heating, need:

   o   A massive central planet (Tidal forces depend on mass)
   o   A moon orbiting close to the massive planet (Tidal forces depend on distance).
   o   Another moon in resonance with the inner moon. (Need eccentric orbit in order to keep distance between inner moon and planet changing)

Io's volcano Tvashtar
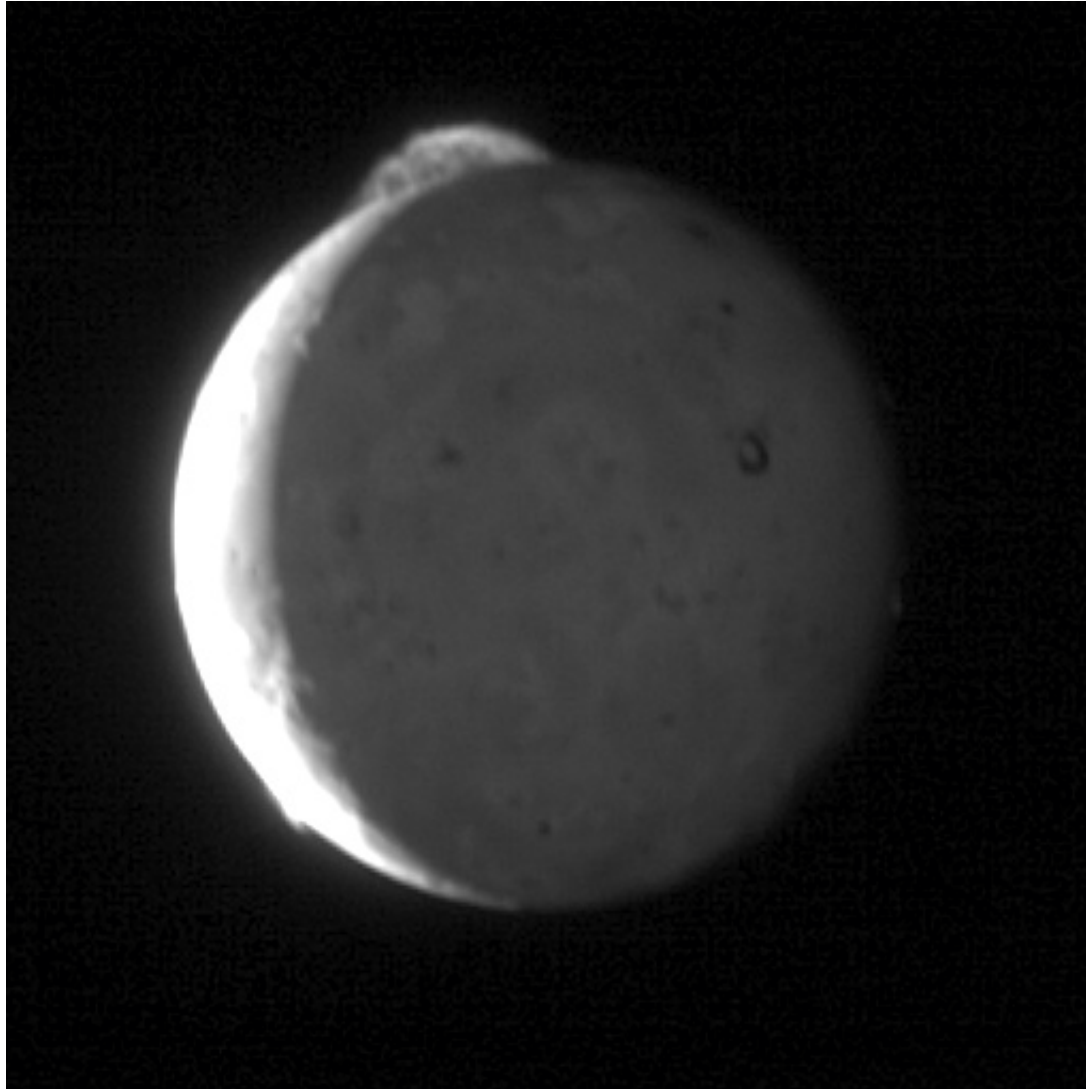
Cooling timescale for Jovian planets

o   Heating during accretion is likely to be the larges heat source for planets.

o   The gain in energy due to accreting from a distance r is over a time dt is:

$$E_{acc} = \left(\frac{GM(r)}{r} - \sigma(T^4(r) - T_0^4) \right) dt$$

o   If accretion is rapid, much of heat of accretion is stored inside planet:

$$\frac{GM(r)}{r} \rightarrow C_P M \Delta T$$

o   If planet cools by radiating energy, it luminosity is: $L = 4\pi R^2 \sigma(T_e^2 - T_{eq}^2)$

o   The rate of change of the mean internal temperature is therefore,

$$\frac{dT}{dt} = \frac{L}{C_V M}$$

where C_V is the specific heat at constant volume (use C_V as assume adiabatic).

Cooling timescale for Jovian planets

o The cooling time can therefore be estimated via $\quad \Delta t \approx \dfrac{\Delta T M C_V}{L}$

o Works well for Jupiter and possibly Saturn, where adiabatic heat transfer holds.

o For Uranus and Neptune, equation for thermal evolution can be written:

$$Cf \frac{dT_e}{dt} = -(T_e^4 - T_{eq}^4)$$

where C is a constant that characterises the thermal inertia of the interior, and f is the fraction of the internal heat reservoir that gives rise to the observed luminosity.

o Drop in temperature for Uranus and Neptune over age of Solar System is ~200 K, which is small compared to larger gas giants (see de Pater and Lissauer, *Planetary Sciences,* for further details).

Energy budgets for Jovian planets

o Observations show Jupiter, Saturn and Neptune radiate more energy into space than they receive from the Sun (for some reason Uranus does not) => must have an internal energy source.

| Planet	Ratio radiated to solar absorbed	Internal Power (Watts)
Jupiter	1.67 ± 0.08	4×10^{17}
Saturn	1.79 ± 0.10	2×10^{17}
Uranus	< 1.4	$< 10^{15}$
Neptune	2.7 ± 0.3	3×10^{15}

o Since the Jovian planets are principally composed of gas and ice, radioactive heating will not be important.

o Energy source is gravitational energy release through shrinking.

Energy generation for Jovian planets

o Consider a simple core plus envelope model of a Jupiter-like planet of total mass M_P.

o If the radius of the envelope changes from R_i to R_f, conservation of energy requires that

Envelope (M_{env})

R_i

Core (M_{core})

$$\Delta K - \frac{GM_{core}M_{env}}{R_f} = -\frac{GM_{core}M_{env}}{R_i}$$

where DK is the (kinetic) energy produced by the contraction of the envelope (change in PE)
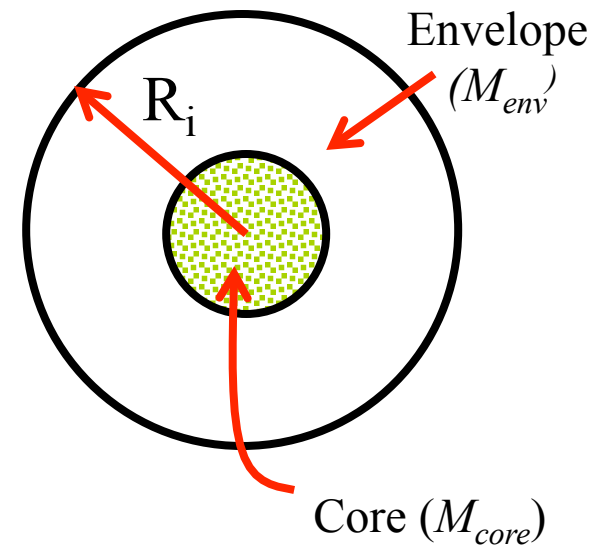
o Therefore,

$$\frac{1}{R_f} - \frac{1}{R_i} = \frac{\Delta K}{GM_{core}M_{env}}$$

Energy generation for Jovian planets

o   For Jupiter:

  o $M_{core} = M_{env} = M_{Jupiter} / 2 = 9.5 \times 10^{26}$ kg
  o $\Delta K = 4 \times 10^{17}$ Watts
  o $R_i = 7.1 \times 10^7$ m (present radius)

o   If we set $R_f = R_i + \Delta R$, then to generate the observed ΔK, Jupiter has to shrink by an amount:

$$\Delta R = -1 \text{ mm / yr}$$

o   This rate of change in the radius amounts to 1 km in a million years - a change that is far to small to measure.

o   Hence the excess energy radiated by Jupiter into space can be easily accommodated by small amounts of shrinkage in its gaseous envelope.